A new Measuring Criterion of the Performance of the Electromagnetic Flowmeter

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Abstract:

The weight function prescribing the sensitivity of the electromagnetic flowmeter (EMF) to the changes in the velocity profiles must be as much as possible uniformly distributed through the measuring volume. The most common used criterion of the weight function distribution is a statistical quantity (ε criterion) which deals with only the axial component of the weight vector.

In the present work an attempt to introduce more revealing and accurate criterion to the EMF performance was studied. The curl of the weight function vector over the measuring volume has been considered and formulated (and termed as ε_c) in such a mathematical expression that takes into account the contributions of the three components of the weight vector regardless of the geometry of cross-sectional area of the flow. In addition, a numerical solution of a previously defined criterion (ε_λ) is presented here for a first time in order to compare the validity of the newly introduced criterion.

The results showed that the present new criterion ε_c is closely harmonious with the previously defined criteria ε and ε_λ in the conventional flow cases. The results and the configuration of the formula of the present criterion which is independent of the flow cross-sectional are led us to conclude that is more reliable and applicable than other existing criteria.

معيار جديد لأداء مقياس الجريان الكهرومغناطيسي

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ملخص البحث:

إن دالة الوزن(weight function) الواصفة لحساسية مقياس الجريان الكهرومغناطيسي للتغيرات في منحني السرعة يجب أن تكون قدر الأمكان موزعة بالتساوي خلال حجم القياس. المعيار الأكثر إستعمالاً لتوزيع دالة الوزن هو معيار ε وهو كمية إحصائية تتعامل مع المركبة المحورية لمتجه دالة الوزن فقط.

تم في هذا البحث دراسة محاولة إيجاد معيار اكثر دقه وكاشف لخفايا أداء مقاييس الجريان الكهرومغناطيسية. لقد تم الأعتماد على حساب التفاف (curl) متجه دالة الوزن (وقد اصطلحنا عليه ε_c) وتركيبه في تعبير رياضي بحيث يأخذ بنظر الاعتبار إسهامات مركبات متجه دالة الوزن الثلاث بغض النظر عن شكل مساحة مقطع الجريان. بالأضافة الى ذلك، تم ولأول مرة، ايجاد حل عددي لمعيار معرف مسبقاً (ε_λ) وذلك لغرض التثبت من مدى امكانية تطبيق المعيار المقترح من خلال عملية المقارنة.

بينت نتائج الحسابات بأن المعيار المقترح متوافق مع المعايير المطروحين مسبقاً (ε and ε_λ) في حالات جريان متعارفة. كما و إن النتائج و التركيبة الرياضية للمعيار المقترح التي لا تعتمد على شكل مساحة مقطع الجريان قادتنا إلى الأستنتاج بانه الأكثر جدارة و التكيف للتطبيق في جميع انواع الجريانات من المعايير المتعارفين.

1.Introduction

Electromagnetic flowmeters are suitable for measuring a wide variety of liquids having a lower electrical conductivity equal to that of distilled water ($4x10^{-6}$ S/m); its accuracy is largely unaffected neither by the changes in environmental conditions nor to the liquid properties [1]. It is the best instrument in the transient flow measurement owing to its very high response. It operates according to the principle of Faraday law of electromagnetic induction. Its conventional form (Fig.1) consists of an insulating cylindrical flow channel (about 3 diameters in length) of circular cross-sectional, in the wall of which are fitted two small diametrically opposed electrodes, their surfaces in contact with the flowing liquid. A suitable external magnetic field is imposed perpendicular to both channel axis and the diameter line joining the two electrodes. An electric potential is produced by the movement of the liquid. The electric potential is picked up by the electrodes, amplified and recorded as a measure of the flow rate through the channel. The main drawback of this flowmeter is that the output signal may be sensitive to any changes in the velocity profile for the same amount of flowrate. This sensitivity could be minimized via a suitable design of geometry(ies) of the magnet and/or electrodes. The theoretical

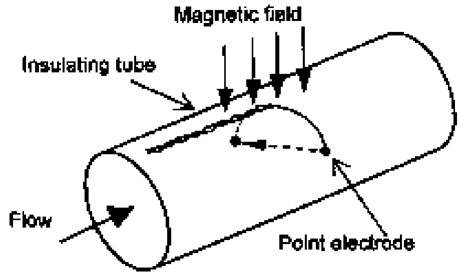

Fig.1 Principle of conventional electromagnetic flowmeter

expression of the induced potential is given by Bevir [2] as a volume integral of the dot product of the liquid velocity vector $\boldsymbol{v}$ and a weight vector $\boldsymbol{W}$ as follow:

$$\Delta U = \int_{\tau} \boldsymbol{v}.\boldsymbol{W} d\tau \qquad (1)$$

Where $\boldsymbol{W}$ is given by:

$$\boldsymbol{W}=\boldsymbol{B}\times\boldsymbol{J} \qquad (2)$$

Here $\boldsymbol{B}$ is the magnetic field vector and $\boldsymbol{J}$ is the virtual current vector (hypothetically, $\boldsymbol{J}$ is the current density that is defined as the unit current that moves between two electrodes with no liquid motion). τ is the effective flowmeter volume, by effective we mean the volume at its two ends both $\boldsymbol{B}$ and $\boldsymbol{J}$ are decay to zero.

The degree of flowmeter influence by the velocity variation is conventionally evaluated depending on the z-component of the weight vector, where in most applications, the flow is assumed to be rectilinear i.e. $v=v_z$, $v_r= v_\theta=0$, then the integral of equation (1) is approximated to:

$$\Delta U = \int_0^{2\pi}\int_0^{R_o} W_{zi}(r,\theta)\ v_z(r,\theta) r dr d\theta \qquad (3)$$

Where R_o is the internal channel radius and $W_{zi}(r,\theta)$ is the integrated rectilinear weight function along the flowmeter effective axis;

$$W_{zi}(r,\theta) = \int_{-\infty}^{+\infty} W_z(r,\theta,z)\ dz \qquad (4)$$

By directing the insight into the elements of the integral of equation (3), it is possible to make the flowmeter signal as a function to the flowrate only by extracting $W_{zi}(r,\theta)$ out of the integral sign i.e. making it constant through the flowmeter cross-sectional area.

2.The conventional measuring criteria of the electromagnetic flowmeters

The most common used criterion of the weight function distribution is the ε measure which has been formulated by Hemp in 1975 [3]. It is a statistical measure of the root mean square variation of the integrated rectilinear weight function with an average value over the flowmeter cross section.

$$\varepsilon = \left[\frac{1}{A}\int_0^{2\pi}\int_0^{Ro}\frac{\left(W_{zi}(r,\theta)-\overline{W_{zi}}\right)^2}{\overline{W_{zi}}^2} r dr d\theta\right]^{1/2} \qquad (5)$$

Where $\overline{W_{zi}} = \frac{1}{A}\int_A W_{zi}(r,\theta)\ dA$

A slight modification was made by AL-Khzraji [4] to the ε criteria of equ. (5), namely the root mean square was removed, hence:

$$\varepsilon = \frac{1}{A}\int_0^{2\pi}\int_0^{Ro}\left|\frac{W_{zi}(r,\theta)}{\overline{W}_{zi}} - 1\right| r dr d\theta \qquad (6)$$

In order to inspect the effect of the real disturbed velocity distribution such as swirl flow, Hemp and Wyatt [5] introduced another alternative criterion including the contribution of the all components of the weight vector as follow:

$$\varepsilon_\lambda = \left[\frac{R_o^3\int_\tau (W - \nabla\lambda)^2\ d\tau}{\int_\tau Wz\, d\tau}\right]^{1/2} \qquad (7)$$

Where λ is Lagrangian multiplier and is a function depending on the flowmeter geometry (as will be explained later in section 4). Wz is the z-component of the weight vector. However, all the three criteria above tend to zero when the flowmeter performance becomes "ideal" i.e.

insensitive to velocity profile which is impossible to be attained in practice.

Many various studies had been reported to improve the flowmeter sensitivity to changes in the velocity profile basing on minimum value of ε criterion defined in equ. (5) [3,6-7,9] and on the ε criterion defined in equ. (6) [4,8,11]. But the literature survey has shown no published study uses ε_λ in spite of its more generality, this may be attributed to the requirement of extra solution of λ inside the flowmeter volume which in turn increases the solution efforts.

Recently, there is a pronounced interest in the use of the electromagnetic flowmeters in the flow metering of two-phase flow regimes [10,12]. In this case the cross sectional area is no longer be constant along the flowmeter z-axis (Fig.2), therefore both ε criteria of equ. (5) and (6) become impossible to be used as they contain averaged quantities over a uniform flow cross-sectional area and also integrations of W_z along lines parallel to the z- axis over the flowmeter length. This meaning appears in studies [10] and [12], where the evaluation of the flowmeter sensitivity to velocity profile did not be included.

Hence in the present work we investigate into the formulation of an alternative comprehensive measuring criterion takes into account the three

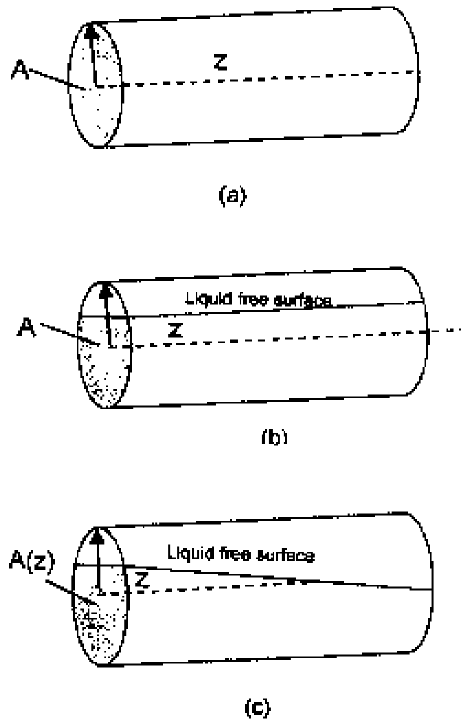

Fig.2 Three types of flow geometry (a) fully filled flow (b) partially filled flow with constant cross- sectional area and (c) partially-filled flow with variable cross sectional area. Z is the axial coordinate.

components of $\boldsymbol{W}$ and irrespective to a cross-sectional area. In addition, a numerical solution of λ and hence data for ε_λ criterion are to be presented here for a first time.

3. A proposed measuring criterion of electromagnetic flowmeters

It was shown in the early literature that the 'ideal' flowmeter, if exists, could be attained if some constraints are imposed on to the weight vector. Bevir 1970 [2] used the facts that there is no flow through the flowmeter wall and $\boldsymbol{W}$ approaches zero up- and down-stream of the flowmeter effective volume. Accordingly, he showed that the sufficient and necessary condition is that the curl of the weight vector is zero everywhere in the flowmeter effective volume (Provided that the liquid is incompressible);

$$\nabla \times W = 0 \tag{8}$$

He pointed out that this condition can be verified only in a certain (but not practical) EMF geometry. He (and other researchers) did neither implement nor formulate this condition. Hence, in this study, attention is focused to interpret the above condition in a dimensionless formula that sustains comprehensive information about the EMF performance. For this purpose, the curl of the weight vector was inserted in more than one mathematical formula. The most stable and quiet level formula among them was chosen and termed as *curl measure* (ε_c). Its final form and formulation details are described as follow;

$$\varepsilon_c = \frac{R_o^4 \int_{\tau_o} \nabla \times W \, d\tau_o}{\tau_o \int_{\tau_o} W_z d\tau_o} \tag{9}$$

The two integrals are taken over the liquid occupying volume of the flowmeter, τ_o. The integral in the denominator ensures that the new criterion free of weight vector units and in the same time determines the flowmeter response to a rectilinear velocity profile. The existing of τ_o in the denominator is very important for two reasons. The first reason is that most of the numerical solutions are achieved by taking a symmetric portion of the whole domain in order to reduce time and to increase accuracy of the solution. However, if τ_o not exists there this mean that the value of ε_c when solving the whole domain, for example, is twice times of its value when solving one-half of the domain if the geometrical symmetry exists. The second reason which is given more efforts in the present study is that the magnitude of $\int_{\tau_o} \nabla \times W d\tau_o$ will has smaller value when the metered liquid not fills all the flowmeter volume as in partially-filled and two-phase flows. This reduction is no longer implies to the improvement of the flowmeter performance against the velocity distribution. Hence, dividing by τ_o will remove this problem. The R_o^4 ensures the dimensionless form of ε_c.

It is worthwhile to emphasize that the present criterion frees of any squared error as in other criteria (equs.(5), (6) and (7)). We expect also that our criterion is more accurate as it involve the contribution of the three components of the weight vector and more reliable as it frees of errors resulting from solving any additional variables as in solving λ of equ.(7) which will be explained in the next category.

4. Evaluation test of the proposed criterion

The calculation of ***W*** is carried out by solving ***J*** and ***B*** in same lattice points of the numerical solution mesh. The present numerical solution was achieved using a uniform finite differences approach. In the case of uniform electrical conductivity, ***J*** is expressed as;

$$J = -\nabla G$$

Which is possible as $\nabla \times J = 0$ (Maxwell's equation) and then G is found by solving Laplacian operator of G in cylindrical coordinates with its appropriate boundary conditions;

$$\nabla^2 G = 0 = \frac{1}{r}\frac{\partial G}{\partial r} + \frac{\partial^2 G}{\partial r^2} + \frac{\partial^2 G}{\partial z^2} + \frac{1}{r^2}\frac{\partial^2 G}{\partial \theta^2}$$

Due to ***B***, assuming that the applied magnetic field is unaffected by the 'small' induced current, thus;

$B = -\nabla F$ and F is found like G (for a given magnetic field geometry). These calculations are necessary and conventional in studying the performance of a newly designed electromagnetic flowmeter. On the other hand, the λ concept was firstly introduced by Hemp and Bevir [5] but a detailed solution is not presented yet in the literature. So that in the following, the solution of λ in the same cylindrical lattice points is achieved to achieve a comparisons among the three criteria measures. The governing equation of λ is:

$$\nabla^2 \lambda = 0 = \frac{1}{r}\frac{\partial \lambda}{\partial r} + \frac{\partial^2 \lambda}{\partial r^2} + \frac{\partial^2 \lambda}{\partial z^2} + \frac{1}{r^2}\frac{\partial^2 \lambda}{\partial \theta^2}$$

With the boundary condition $\frac{\partial \lambda}{\partial n} = W_n$ on the wall, were n here is direction along the normal to the wall.

Due to geometrical symmetry, the numerical solution was achieved for one-forth of the overall flowmeter volume with the following boundary conditions (which are illustrated in Fig.3);

1) G=F=0 at z=L (flowmeter end)

2) $\frac{\partial G}{\partial z} = \frac{\partial F}{\partial z} = 0$ at z=0 (symmetry plane)

3) G=1 on the electrode surface

4) $\frac{\partial G}{\partial r} = 0$ on the insulating flowmeter wall

5) $\frac{\partial G}{\partial n} = 0$ on the liquid free surface (if exists)

6) G=0 on the symmetry plane θ=0 and θ=π.

7) F=+f(θ) on the north pole piece of the magnet and F=-f(θ) on the south one.

Where $f(\theta)$ represents the distribution of F potential that gives either optimum or uniform magnetic distributions. The optimum magnetic field is series of F's which are obtained using a numerical optimization procedure listed in [9]. The uniform magnetic field is $B=B_y$=constant over the measuring volume, merely.

8) F=0 on the insulating flowmeter wall

9) $\frac{\partial F}{\partial \theta}=0$ on the symmetry plane θ=0 and θ=π.

10) $\frac{\partial \lambda}{\partial r}=W_r$ on the flowmeter wall

11) $\frac{\partial \lambda}{\partial r}=0$ on the electrode surface.

12) $\frac{\partial \lambda}{\partial z}=0$ at z=0 and at z=L

13) λ=0 on the symmetry plane θ=0 and θ=π.

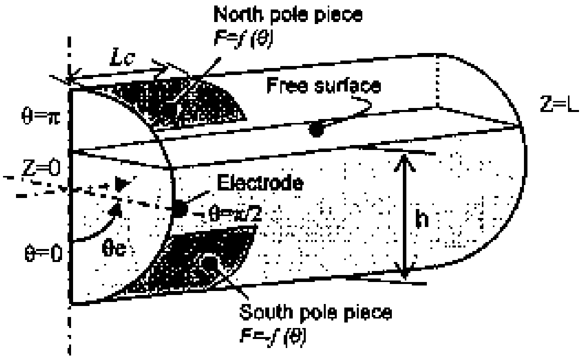

Fig. 3 One-forth of the original flowmeter volume representing the domain of the numerical solution

5. Results and discussions

To check the validity of the present criterion and comparing it with the other two criteria many solutions were given for different geometries of magnets (uniform and optimum) together with point electrodes fixed at the mid length of the flowmeter but for different angular positions. Two flow situations were considered, fully-and partially-filled flows. The most suitable number of finite deference nodes that give less percentage solution error (about 0.1%) and faster convergence was found to be 2176 nodes distributed uniformly over the symmetric domain of solution (Fig.4). The singularities laying on the axial line r=0 were obviated by shifting all nodes one-half grid in the radial direction. Because of the solution of λ depends on the radial component of the weight vector (W_r), hence its solution must be initiated as soon as finishing the solution of both G and F potentials. This is may be the reason why in most flowmeter geometries the accuracy of λ solution is less than that of G and F solutions.

For comparison purpose, the built program (which is written in Fortran) was adapted to generate a uniform magnetic field, $B=(0,B_y,0)$, but the present descritization is based on cylindrical coordinates, hence by

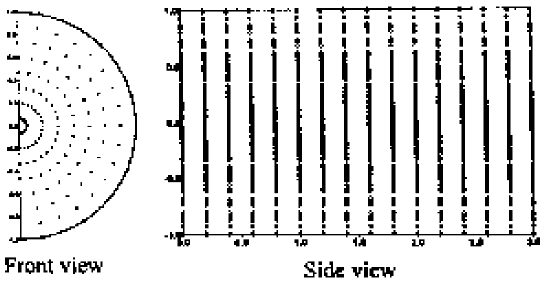

Fig.4 Lattice points of the numerical solution

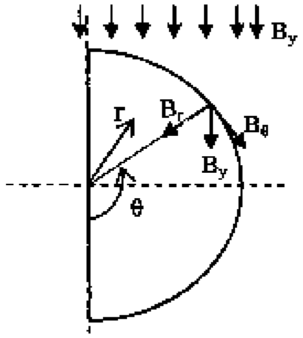

Fig.5 Interpretation of the uniform Magnetic field in cylindrical coordinates

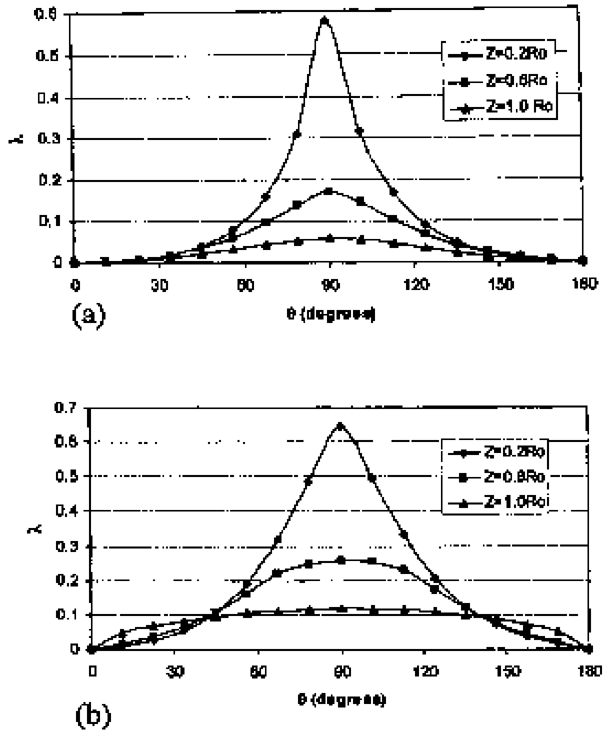

Fig.6 Distribution of λ over the flowmeter wall, r=Ro for (a) uniform magnetic field and (b) optimum magnetic field

referring to Fig.5 the following relations are obtained:

$\mathbf{B_r=B_y\cos(\pi-\theta)}$, $\mathbf{B_\theta=B_y\sin(\pi-\theta)}$, $\mathbf{B_z=0}$

The distributions of λ over the flowmeter wall are presented in Fig.6 (a) for uniform magnetic field and Fig. 6 (b) for optimum magnetic field. It is clear that how λ values decay when one moves toward the flowmeter end far from the flowmeter center where the point electrode is fixed at $z=0$, $\theta=\pi/2$.

For the uniform magnetic filed together with point electrodes geometry, Hemp and Wyatt [5] reported an approximate value for the ε_λ criterion that is 0.63 and they mentioned that this flowmeter is known to be highly sensitive to velocity distributions. Hence the value of 0.63 could be serves as an upper value with which values of ε_λ for other (improved) flowmeters may be compared. However, the present numerical solution gives ε_λ =0.577 for the same geometry of [5], hence the percentage difference in ε_λ values between the present numerical study and that of Ref.[5] (which is approximately calculated) is -8.4%.

Turning now to the optimum magnetic field distribution which was proved that is more efficient in improving the flowmeter performance [9,11].

It is well known that the best position of point electrodes in conventional electromagnetic flowmeter is what joined by a line of one diameter and this line is perpendicular to the center of the magnet (see Fig.3), nevertheless, many angular positions less and greater than this conventional position were checked in order to excite weight function non-uniformities and then examine the behavior of ε_c criterion and comparing it with the other two criteria. Figures (7 a to c) illustrate this comparison. The present ε_c criterion together with the other two ε and ε_λ have symmetrical behavior around the conventional flowmeter design where they indicate that the weight function non-uniformity, for both optimum and uniform magnetic field geometry, increases when the point electrode is moved below or above its conventional position $\theta e=\pi/2$, and this evidence is true as the liquid fills all the flowmeter volume. Unlike with ε and ε_λ, the ε_c criterion indicate to a better (less) value in the uniform than optimum magnetic field. The reason of this is that in uniform magnetic field, one of the ***B*** components is assumed to be zero (Bz=0) and this lead to a lesser value

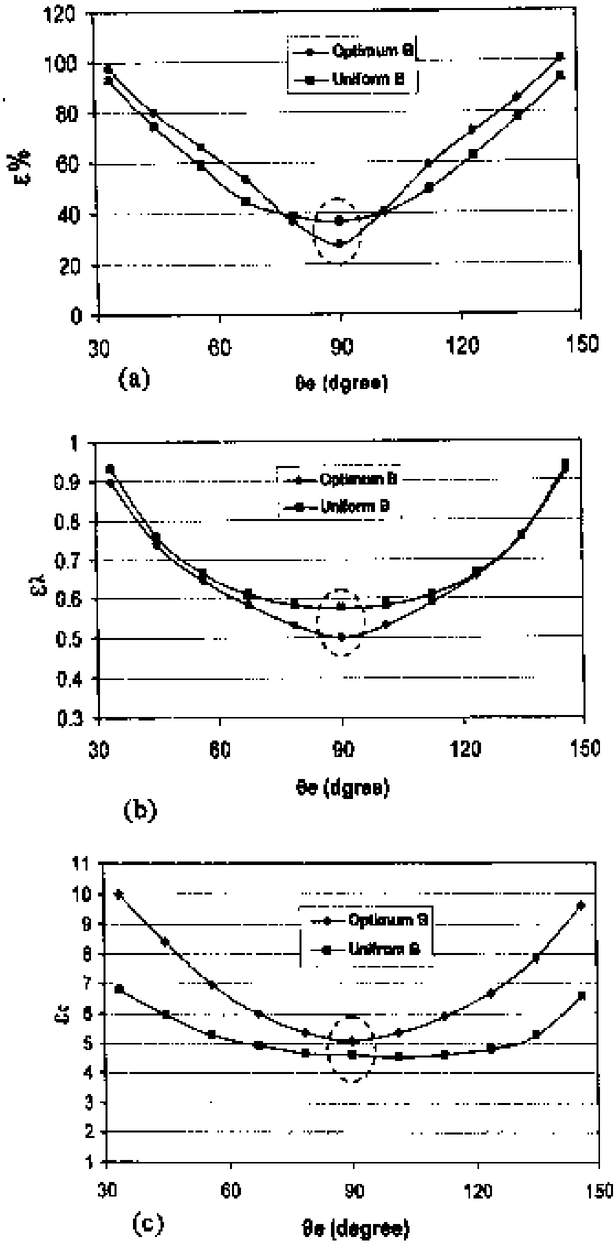

Fig.7 Values of three formulas of ε's for two types of magnetic field. The liquid fully fills the flowmeter volume.

of $\int_{\tau_o} \nabla \times W d\tau_o$. But this is no longer mean better flowmeter performance in practice as the uniform magnetic field doesn't contain the contribution of the three components of the metered liquid velocity vector on the flowmeter output signal.

It is worthwhile here to point out that in the contoured regions in Figs 7 a-c, the curves of optimum magnetic field is steeper than that of uniform magnetic field. This is attributed to that the optimum magnetic filed is already found for a point electrode geometry fixed at this contoured position.

Turning now to check and compare the ε_c criterion for different flow geometry, namely partially-filled pipe flow. Only the optimum magnetic field was considered in this case but for three different electrode angular positions. The results of ε's values are presented in Figs 8 a-c. For each electrode position the data in these figures are restricted with the level of liquid over the horizontal line joining each electrode pair, where the electromagnetic flowmeter becomes unusable when the liquid level doesn't be in contact with the inner electrode surface. The train of ε_c and ε_λ are greatly harmonious while a pronounced discrepancy appears when comparing with ε train (Fig. 8a). This discrepancy refers to that both ε_c and ε_λ deal with the three components of the weight vector $\boldsymbol{W}$ rather than the axial component W_z only as in ε criterion. Also, when a liquid free surface exists, the weight vector components are greatly influenced by the liquid level.

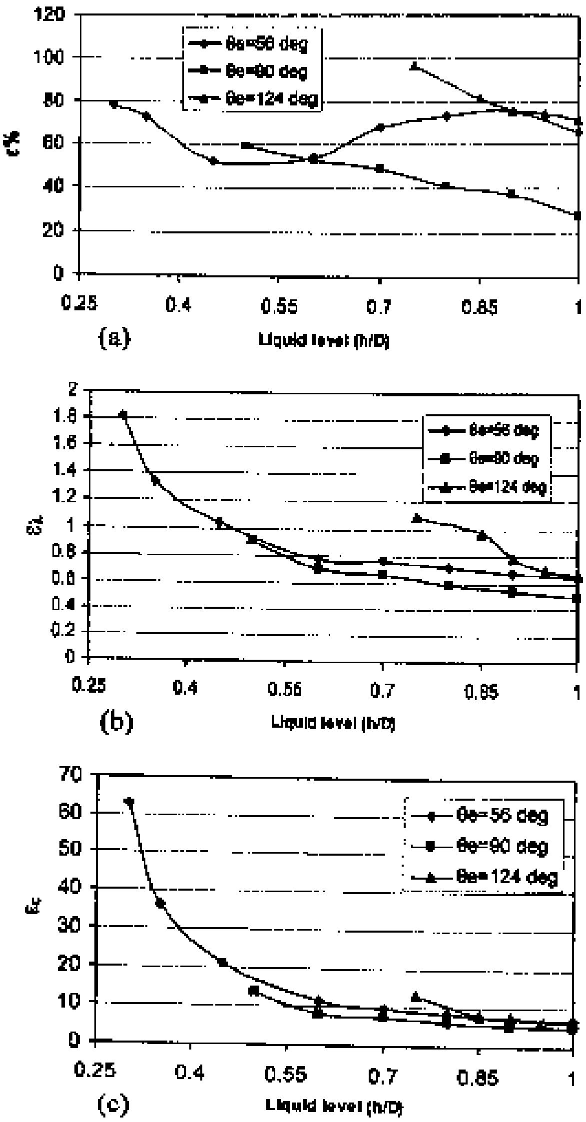

Fig.8 Values of three formulas of ε's for optimum magnetic field. The liquid partially fills the flowmeter volume.

6.Conclusions

A new mathematical formula for judging the electromagnetic flowmeter sensitivity against the velocity distributions was introduced, solved and compared with the existing other formulas. A numerical solution for the previously defined ε_λ

criterion measure was achieved and presented here for a first time for the purpose of verifying the validity of the present ε_c criterion measure. The configuration of ε_c criterion measure, which is formulated to be irrespective of the cross-sectional area of the metered liquid, was found to be (i) harmonious with the other two measures in fully-filled pipe flow and (ii) the most stable in partially-filled pipe flow . This led us to conclude that the new proposed criterion measure ε_c is the most generality criterion for measuring the weight function non-uniformity in any complicated flow geometry like partially-filled pipe flow with irregular free surface. Such complex flow geometry doe's not studied hitherto as it require some hard numerical treatments for the solution of G potential.

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Notations

A	Cross-sectional flow area	W_z	Axial component of $\boldsymbol{W}$
$\boldsymbol{B}$	Magnetic field vector	$W_{zi}(r,\theta)$	Rectilinear weight function
F	Magnetic field scalar potential	$\overline{W}_{zi}$	Average of $W_{zi}(r,\theta)$ over the flow cross section area
G	Virtual current scalar potential	$r dr d\theta$	Cylindrical coordinates
h	Liquid level	ΔU	Flowmeter output signal
$\boldsymbol{J}$	Virtual current density vector	ε	Criterion of the weight function non-uniformity
L	Half flowmeter length	ε_λ	Criterion of the weight vector non-uniformity
Lc	Half magnet length	ε_c	Present criterion of the weight vector non-uniformity
Ro	Flowmmeter-inside radius	λ	Lagranagian multiplier used in calculating ε_λ
$\boldsymbol{n}$	Normal vector	θ_e	Angular electrode position
$\boldsymbol{v}$	Liquid velocity vector	τ	Volume of the measuring section
$\boldsymbol{W}$	Weight vector	τ_0	Effective volume of the measuring section